Digest 8

(A compilation of emailed homework questions, answered around Tuesday.)

Question. (From a student): In section 6.3 Question 21, the problem asks for the total area bounded between the curve $y(x) = x^3 - x$ and the x-axis. How do I do this?

Answer. Note that $y(x) = 0$ only when $x = -1, 0, 1$, and the curve $y(x)$ crosses the x-axis at these three points (since $y'(x) \neq 0$ at these three points.) So, the area bounded between the curve and the x-axis consists of two regions: the region above $[-1, 0]$ and the region below $[0, 1]$. So, the requested area is $\int_{-1}^{0} y(x)dx - \int_{0}^{1} y(x)dx$.

Question. (From a student): In section 6.4 Question 4, it says the demand curve is $p = 100 - q^2$ and the supply curve is $p = 2 + q^2$. It then asks for the producer surplus when the market is at equilibrium. How do I do this?

Answer. I think I assigned this question accidentally, so I am just going to tell you how to do it.

First, equilibrium occurs when supply and demand are equal. (Here q denotes the quantity of the good, and the p values denote market prices.) So, we set supply equal to demand and solve for q to get $100 - q^2 = 2 + q^2$, so that $2q^2 = 98$, i.e. $q^2 = 49$ so that $q = 7$ (since a negative quantity of good is not meaningful). When $q = 7$ the equilibrium price is $100 - q^2 = 51$.

Okay, so now we know $q = 7$ is the equilibrium. When $0 < q < 7$, demand exceeds supply. Producer surplus is defined to be the area *above* the supply curve that is below the equilibrium price (which is 51 in this case). So, in this case producer surplus is

$$\int_{0}^{7} (51 - (2 + q^2))dq = \int_{0}^{7} (49 - q^2)dq = 49(7) - (1/3)7^3 = 7^3(1 - 1/3) = 7^3(2/3) \approx 228.67.$$

Question. (From a student): In section 5.5 question 10, it says the marginal cost is $C'(q) = \frac{600}{(.3)q+5}$. Part (c) asks for the marginal profit on the 31st bicycle if bikes are sold for \$205. How do I do this?

Answer. The marginal profit on the 31st bicycle is the marginal revenue (205) minus the marginal cost of producing the 31st bicycle. The online homework system interprets the marginal revenue of producing the 31st bicycle as $C'(30)$, so the answer it wants is $205 - C'(30)$. I think this is a bit questionable since you could also interpret $C'(31)$ as the marginal revenue from the 31st bicycle, but this is far enough away from the answer it can actually register as incorrect.