Math 118, Section 39441, Fall 2018, USC   Instructor: Steven Heilman

Name: _____   USC ID: _____   Date: _____

Signature: _____.
(By signing here, I certify that I have taken this test while refraining from cheating.)

Mid-Term 1

This exam contains 7 pages (including this cover page) and 5 problems. When the exam begins, check for missing pages. Enter all requested information on the top of this page.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- You have 50 minutes to complete the exam, starting at the beginning of class.

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.

- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

- If you need more space, use the back of the pages; clearly indicate when you have done this. Scratch paper appears at the end of the document.

Do not write in the table to the right. Good luck!^{*a*}

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

_____

1. (a) (5 points) Evaluate the following limit. If the limit does not exist, write DNE.

$$\lim_{x \to \infty} \left(\sqrt{x^2 + x + 1} - \sqrt{x^2 + x} \right).$$

(b) (5 points) Is there a real number a such that the following limit exists?

$$\lim_{x \to 1} \frac{3x^2 + ax + a + 1}{(x - 1)(x + 1)}$$

If so, find the value of a and the value of the limit.

2. Label the following statements as TRUE or FALSE. Then, briefly **justify your answer**.

   (a) (3 points) If a function is continuous at 0, then $f'(0)$ exists.

$$\text{TRUE} \qquad \text{FALSE} \quad \text{(circle one)}$$

   (b) (4 points) Let $f \colon \mathbf{R} \to \mathbf{R}$. If $f(0) = -1$ and $f(1) = 1$, then there is some $x \in [0, 1]$ such that $f(x) = 0$.

$$\text{TRUE} \qquad \text{FALSE} \quad \text{(circle one)}$$

   (c) (3 points) If $b > 0$, then $\frac{d}{dx} b^x = b^x$.

$$\text{TRUE} \qquad \text{FALSE} \quad \text{(circle one)}$$

3. (10 points) Find the equation of all tangent lines to the curve $y = e^{-x^2}$ that are parallel to the line $y = 3$.

4. (10 points) Let $f(x) = x^3$. Using the **definition of the derivative**, show that

$$f'(x) = 3x^2.$$

5. (10 points) Let f be a function such that $f(1) = 2$, $f(2) = 3$, $f'(1) = 4$, $f'(2) = 9$ and $f'(3) = 6$. Define
$$F(x) = \frac{1}{f(xf(x))}.$$
Find $F'(1)$.

(Scratch paper)