Please provide complete and well-written solutions to the following exercises.

No due date, but the quiz in Week 3 in the discussion section (on September 6) will be based upon this homework.

Q3: Quiz 3 Problems

Exercise 1. Compute

$$\lim_{y\to\infty} \frac{4y^5+5}{(y^2-2)(2y^2-1)}.$$

Exercise 2. Find all horizontal and vertical asymptotes of the following functions:

- $y = 1/x$.
- $y = \sqrt{x^2+x+1} - \sqrt{x^2+x}$.

(A vertical asymptote occurs at a when any one-sided limit of y at a is ∞ or $-\infty$. A horizontal asymptote occurs if $\lim_{x\to\infty} y(x)$ exists, or if $\lim_{x\to-\infty} y(x)$ exists.)

Exercise 3. Show that the function $f(x) = x^3 - x - 1$ has a zero between -1 and 2.

Exercise 4. Let $g(x) = x^{2/3}$.

- Show that $g'(0)$ does not exist.
- If $a \neq 0$, find $g'(a)$, using the definition of the derivative.
- Show that $y = x^{2/3}$ has a vertical tangent line at $x = 0$. (A vertical tangent line occurs at a when $\lim_{x\to a} |g'(x)| = \infty$.)
- Demonstrate the vertical tangent line by graphing $y = x^{2/3}$.

(Hint for second item: Recall that $c^3 - d^3 = (c-d)(c^2+cd+d^2)$. Using $c = (a+h)^{2/3}$ and $d = a^{2/3}$, we get the formula $(a+h)^2 - a^2 = ((a+h)^{2/3} - a^{2/3})((a+h)^{4/3} + (a+h)^{2/3}a^{2/3} + a^{4/3})$. This might be useful when you write the difference quotient.)

Exercise 5. Suppose a baseball is thrown vertically upward with a velocity of 80 ft/s. Then its height after t seconds is $r(t) = 80t - 16t^2$.

- What is the maximum height reached by the baseball?
- What is the velocity of the ball when it is 96 feet above the ground on the way up? And on the way down?

Exercise 6. Suppose the curve $y = x^4 + ax^3 + bx^2 + cx + d$ has a tangent line when $x = 0$ with the equation $y = 2x + 1$ and a tangent line when $x = 1$ with equation $y = 2 - 3x$. Find the values of a, b, c and d.