Math 167, Winter 2016, UCLA Instructor: Steven Heilman

Name: _____ UCLA ID: _____ Date: _____

Signature: _____.
(By signing here, I certify that I have taken this test while refraining from cheating.)

Mid-Term 1

This exam contains 7 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page.

You may *not* use your books, notes, or any calculator on this exam. You are required to show your work on each problem on the exam. The following rules apply:

- You have 50 minutes to complete the exam, starting at the beginning of class.
- **If you use a theorem or proposition from class or the notes or the book you must indicate this** and explain why the theorem may be applied. It is okay to just say, "by some theorem/proposition from class."
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this. Scratch paper appears at the end of the document.

Problem	Points	Score
1	15	
2	20	
3	25	
4	15	
5	15	
Total:	90	

Do not write in the table to the right. Good luck![a]

[a] January 13, 2016,

1. (a) (5 points) Give an example of a game mentioned in class, or the notes, or any course textbook, such that: the first player has a winning strategy. (You only need to mention the game; you do not need to prove anything.)

   (b) (5 points) Give an example of a game mentioned in class, or the notes, or any course textbook, such that: the second player has a winning strategy. (You only need to mention the game; you do not need to prove anything.)

   (c) (5 points) Give an example of a game mentioned in class, or the notes, or any course textbook, such that: both players have a strategy guaranteeing at least a draw. (You only need to mention the game; you do not need to prove anything.)

2. (20 points) Consider the game of Nim, where the game starts with four piles of chips. These piles have $1, 5, 3$ and 15 chips, respectively. Which player has a winning strategy from this position, the first player, or the second? Describe a winning first move. Prove that this move is a winning first move.

3. (25 points) Describe the optimal strategies for both players for the two-person zero-sum game described by the payoff matrix below. That is, at the optimal strategy, with what probability does player I play C, with what probability does player I play D, with what probability does player II play A, with what probability does player II play B?

		Player II	
		A	B
Player I	C	0	1
	D	2	1

Prove that these strategies are optimal.

4. (15 points) Let Y be a random variable such that: $Y = 2$ with probability $1/2$, $Y = 3$ with probability $1/2$.

   Let Z be a random variable such that: $Z = 1$ with probability $1/2$ and $Z = 2$ with probability $1/2$. Assume that Z and Y are independent. What is the probability that: $Y = 3$ and $Z = 2$? What is the expected value of $Y \cdot Z$?

5. (15 points) Explicitly define some function $f\colon [0,1] \times [0,1] \to \mathbf{R}$ such that

$$\min_{y\in[0,1]} \max_{x\in[0,1]} f(x,y) \neq \max_{x\in[0,1]} \min_{y\in[0,1]} f(x,y).$$

Prove that the function f satisfies this property.

(Scratch paper)