Please provide complete and well-written solutions to the following exercises.

Due April 4, in the discussion section.

(This Review Assignment will be collected, but this Review Assignment will not be graded.)

Preliminary Review Assignment

Exercise 1. As needed, refresh your knowledge of proofs and logic by reading the following document by Michael Hutchings: http://math.berkeley.edu/∼hutching/teach/proofs.pdf

Exercise 2. Take the following quizzes on logic, set theory, and functions:

http://scherk.pbworks.com/w/page/14864234/Quiz%3A%20Logic
http://scherk.pbworks.com/w/page/14864241/Quiz%3A%20Sets
http://scherk.pbworks.com/w/page/14864227/Quiz%3A%20Functions

(These quizzes are just for your own benefit; you don't need to record your answers anywhere.)

Exercise 3. Prove the following assertion by induction:

For any natural number n, $1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1)$.

Exercise 4. Prove that the set of real numbers $\mathbf{R}$ can be written as the countable union

$$\mathbf{R} = \bigcup_{j=1}^{\infty} [-j, j].$$

(Hint: you should show that the left side contains the right side, and also show that the right side contains the left side.)

Prove that the singleton set $\{0\}$ can be written as

$$\{0\} = \bigcap_{j=1}^{\infty} [-1/j, 1/j].$$

Exercise 5. Retake at least one of the finals I gave when I taught math 170A:

http://www.math.ucla.edu/ heilman/teach/170afinal.pdf
http://www.math.ucla.edu/∼heilman/teach/170afinalsoln.pdf
http://www.math.ucla.edu/∼heilman/teach/170afinalv2.pdf
http://www.math.ucla.edu/∼heilman/teach/170afinalv2soln.pdf

Exercise 6. Using your favorite computing language, create a plot of 1000 samples drawn from the uniform distribution on $[-1, 1]$. Then, create a histogram of 1000 samples drawn from the standard Gaussian distribution.

(For creating plots like this quickly and easily, I recommend using Matlab, which you can read more about here. The functions `rand` and `randn` would be most relevant. Matlab is a good software for educational purposes, since the syntax is very easy and forgiving, but Matlab is not good for handling large data sets in e.g. industrial applications. When we have the occasional (optional) computer-based exercise, I will always write a solution in Matlab. The Open Source version of Matlab is known as Octave. Also, many statisticians like R.

For more industrial applications, C (and its variants) or JAVA are probably the best.

I don't care what language you use, as long as you can occasionally do computational exercises like this.)