

Name: _____ USC ID: _____ Date: _____

Signature: _____. Discussion Section: _____

(By signing here, I certify that I have taken this test while refraining from cheating.)

Exam 2

This exam contains 8 pages (including this cover page) and 5 problems. Enter all requested information on the top of this page.

You may *not* use your books, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- You have 50 minutes to complete the exam, starting at the beginning of class.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this. Scratch paper appears at the end of the document.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

Do not write in the table to the right. Good luck!^a

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1. Label the following statements as TRUE or FALSE. Make sure to **explain your reasoning**. That is, you should **justify your answer**.

(a) (2 points) There is a random variable X such that

$$\text{Var}(X) = -1.$$

TRUE FALSE (circle one)

(b) (2 points) Let X, Y be random variables. Then

$$\text{cov}(X, Y) \geq 0.$$

TRUE FALSE (circle one)

(c) (2 points) Let X, Y be random variables. Then

$$\mathbf{E}(XY) = (\mathbf{E}X)(\mathbf{E}Y).$$

TRUE FALSE (circle one)

(d) (2 points) Let X be a random variable and let $t > 0$ be a real number. Then

$$\text{Var}(tX) = t^2\text{Var}(X).$$

TRUE FALSE (circle one)

(e) (2 points) Let X, Y be random variables. Then

$$\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y).$$

TRUE FALSE (circle one)

2. Suppose random variables X and Y have joint probability density function

$$f_{X,Y}(x,y) = \begin{cases} xy & , \text{ if } 0 \leq x \leq \sqrt{2} \text{ and } 0 \leq y \leq \sqrt{2}. \\ 0 & , \text{ otherwise.} \end{cases}$$

(a) (2 points) Compute the X marginal f_X . Simplify your answer as best you can.

(b) (2 points) Compute the Y marginal f_Y . Simplify your answer as best you can.

(c) (3 points) Are X and Y independent? Justify your answer.

(d) (3 points) Compute $\mathbf{E}(XY + 1)$. Simplify your answer to the best of your ability.

3. Suppose I flip a fair coin many times over and over again. (A fair coin has probability $1/2$ of landing heads, and probability $1/2$ of landing tails.)

Let X be the number of tails that occur before the first head appears.

Let Y be the number of tails that occur after the first head appears, and before the second head appears.

- (a) (3 points) Give a formula for the probability mass function p_X of X . Simplify to the best of your ability. Justify your answer.

- (b) (3 points) Give a formula for the probability mass function p_Y of Y . Simplify to the best of your ability. Justify your answer.

- (c) (4 points) Give a formula for the joint probability mass function $p_{X,Y}$ of X and Y . Simplify to the best of your ability. Justify your answer.

4. Suppose I roll a fair six-sided die n times. (A fair six-sided die has probability $1/6$ of each of its six faces appearing after each roll.)

Let X be the number of times that the die face labelled “1” appears.

Let Y be the number of times that the die face labelled “2” appears.

(a) (4 points) Compute $\text{Var}(X)$. Simplify as best you can. Justify your answer.

(b) (6 points) Compute $\text{Cov}(X, Y)$. Simplify as best you can. Justify your answer.

5. (10 points) Suppose you have a standard 52 card deck of playing cards. The deck is shuffled, so that each of the $52!$ arrangements of the cards are equally likely to occur.

Compute the expected number of cards you have to draw from the top of the deck before you see a heart appear.

(For example, if the top card on the deck is itself a heart, you had to draw zero cards before you see a heart appear.)

(A standard 52 card deck has 13 hearts and 39 non-heart cards.)

(Scratch paper)