Probability, 407, Fall 2020, USC — Instructor: Steven Heilman

Name: ______________________ USC ID: ______________ Date: ___________

Signature: ______________________.
(By signing here, I certify that I have taken this test while refraining from cheating.)

Mid-Term 2

This exam contains 7 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page.

You may use your books and notes on this exam. You cannot use a calculator or any other electronic device (or internet-enabled device) on this exam. You are required to show your work on each problem on the exam. The following rules apply:

- You have 50 minutes to complete the exam, starting at the beginning of class.
- **If you use a theorem or proposition from class or the notes or the book you must indicate this** and explain why the theorem may be applied. It is okay to just say, "by some theorem/proposition from class."
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this. Scratch paper is at the end of the exam.

Do not write in the table to the right. Good luck![a]

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

[a]October 21, 2020,

1. (10 points) Let X be a binomial random variable with parameters $n = 2$ and $p = 1/4$ so that for all integers k satisfying $0 \le k \le 2$,

$$\mathbf{P}(X = k) = \binom{2}{k} p^k (1-p)^{n-k} = \frac{2!}{k!(2-k)!} (1/4)^k (3/4)^{2-k}.$$

Compute the following quantities:

- $\mathbf{E}X$
- $\mathbf{E}(X^2)$
- $\text{var}(X)$

Simplify your answers to the best of your ability. (As usual, show your work.)

2. (10 points) Let X be a standard Gaussian random variable, so that X has PDF

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}, \qquad \forall\, x \in \mathbf{R}.$$

Compute the following quantities:

- $\mathbf{P}(X = 10)$.
- $\mathbf{P}(X > 0)$.

Simplify your answers to the best of your ability. (As usual, show your work.)

3. (10 points) Let $0 < p < 1$. Let X be a geometric random variable with parameter p, so that, for any positive integer k,

$$\mathbf{P}(X = k) = (1-p)^{k-1}p.$$

In class, we computed $\mathbf{E}X = 1/p$ and $\mathbf{E}X^2$ using a conditioning argument. For example, we showed $\mathbf{E}(X^2|X = 1) = 1$ and $\mathbf{E}(X^2|\, X > 1) = 1 + 2\mathbf{E}X + \mathbf{E}X^2$. We then solved for $\mathbf{E}X^2$ to get $\mathbf{E}X^2 = (2/p^2) - 1/p$.

Using this same conditioning argument (i.e. by conditioning on $X = 1$ and on $X > 1$), compute $\mathbf{E}(X^3)$.

4. (10 points) Let X be binomial random variable with parameters $n = 2$ and $p = 1/2$. So, $\mathbf{P}(X = 0) = 1/4$, $\mathbf{P}(X = 1) = 1/2$ and $\mathbf{P}(X = 2) = 1/4$. And X satisfies $\mathbf{E}X = 1$ and $\mathbf{E}X^2 = 3/2$.

   Let Y be a geometric random variable with parameter $1/2$. So, for any positive integer k, $\mathbf{P}(Y = k) = 2^{-k}$. And Y satisfies $\mathbf{E}Y = 4$ and $\mathbf{E}Y^2 = 32$.

   Let Z be a Poisson random variable with parameter 1. So, for any nonnegative integer k, $\mathbf{P}(Z = k) = \frac{1}{e}\frac{1}{k!}$. And Z satisfies $\mathbf{E}Z = 1$ and $\mathbf{E}Z^2 = 2$.

   Let W be a discrete random variable such that $\mathbf{P}(W = -1) = 1/2$ and $\mathbf{P}(W = 1) = 1/2$, so that $\mathbf{E}W = 0$ and $\mathbf{E}W^2 = 1$.

   Assume that X, Y and Z are all independent. Compute

$$\mathbf{E}(1 + W^{100} + W^{50}XYZ^2).$$

   (You **cannot** assume that W is independent of X, Y, Z.)

5. (10 points) Let $X_1, \ldots, X_n$ be independent standard Gaussian random variables. Let $Y = \min(X_1, \ldots, X_n)$ be the minimum of $X_1, \ldots, X_n$. What is the PDF of Y?

(Scratch paper)