Please provide complete and well-written solutions to the following exercises.

Due August 24, 12PM noon PST, to be uploaded as a single PDF document to Gradescope.

Homework 1

Exercise 1. As needed, refresh your knowledge of proofs and logic by reading the following document by Michael Hutchings: http://math.berkeley.edu/∼hutching/teach/proofs.pdf

Exercise 2. As needed, take the following quizzes on logic and set theory:

http://scherk.pbworks.com/w/page/14864234/Quiz%3A%20Logic
http://scherk.pbworks.com/w/page/14864241/Quiz%3A%20Sets

(These quizzes are just for your own benefit; you don't need to record your answers anywhere.)

Exercise 3. Two people take turns throwing darts at a board. Person A goes first, and each of their throws has a probability of $1/4$ of hitting the bullseye. Person B goes next, and each of their throws has a probability of $1/3$ of hitting the bullseye. Then Person A goes, and so on. With what probability will Person A hit the bullseye before Person B does?

Exercise 4. Suppose you have a car with twenty tires, and the car mechanic removes all twenty tires. Suppose the mechanic now puts the tires back on randomly, so that all arrangements of the tires are equally likely. With what probability will no tire end up in its original position? Give an answer to ten decimal places of accuracy (e.g. your answer could be 0.1234567891). Can you guarantee that these ten decimal places are correct?

Exercise 5. Suppose a test for a disease is 99.9% accurate. That is, if you have the disease, the test will be positive with 99.9% probability. And if you do not have the disease, the test will be negative with 99.9% probability. Suppose also the disease is fairly rare, so that roughly 1 in 20,000 people have the disease. If you test positive for the disease, with what probability do you actually have the disease?

Exercise 6. Suppose I tell you that the following list of 20 numbers is a random sample from a Gaussian random variable, but I don't tell the mean or standard deviation.

5.1715,   3.2925,   5.2172,   6.1302,   4.9889,   5.5347,   5.2269,   4.1966,   4.7939,   3.7127

5.3884,   3.3529,   3.4311,   3.6905,   1.5557,   5.9384,   4.8252,   3.7451,   5.8703,   2.7885

To the best of your ability, determine what the mean and standard deviation are of this random variable. (This question is a bit open-ended, so there could be more than one correct way of justifying your answer.)

Exercise 7. Suppose I tell you that the following list of 20 numbers is a random sample from a Gaussian random variable, but I don't tell you the mean or standard deviation. Also, around one or two of the numbers was corrupted by noise, computational error, tabulation error, etc., so that it is totally unrelated to the actual Gaussian random variable.

$$-1.2045, \; -1.4829, \; -0.3616, \; -0.3743, \; -2.7298, \; -1.0601, \; -1.3298, \; 0.2554, \; 6.1865, \; 1.2185$$
$$-2.7273, \; -0.8453, \; -3.4282, \; -3.2270, \; -1.0137, \; 2.0653, \; -5.5393, \; -0.2572, \; -1.4512, \; 1.2347$$

To the best of your ability, determine what the mean and standard deviation are of this random variable. Supposing you had instead a billion numbers, and 5 or 10 percent of them were corrupted samples, can you come up with some automatic way of throwing out the corrupted samples? (Once again, there could be more than one right answer here; the question is intentionally open-ended.)