Math 446, Fall 2024, USC Instructor: Steven Heilman

Name: _____ USC ID: _____ Date: _____

Signature: _____. Discussion Section: _____
(By signing here, I certify that I have taken this test while refraining from cheating.)

# Exam 2

This exam contains 8 pages (including this cover page) and 5 problems. Enter all requested information on the top of this page.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- You have 50 minutes to complete the exam, starting at the beginning of class.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this. Scratch paper appears at the end of the document.

| Problem | Points | Score |
|---|---|---|
| 1 | 8 | |
| 2 | 10 | |
| 3 | 10 | |
| 4 | 10 | |
| 5 | 10 | |
| Total: | 48 | |

Do not write in the table to the right. Good luck![a]

[a] November 9, 2024, 

1. Label the following statements as TRUE or FALSE. If the statement is true, **EXPLAIN YOUR REASONING**. If the statement is false, **PROVIDE A COUNTEREXAMPLE OR EXPLAIN YOUR REASONING**.

   (a) (2 points) Python raises an exception (i.e. gives an error) when given the command `{[1, 2], 3}`

   TRUE FALSE (circle one)

   [This was repeated from Exam 1]

   (b) (2 points) Python's implementation of training a neural network is deterministic. That is, if we use the command `from tensorflow.keras.models import Sequential`, and then define `neural_model()` to be a neural network, then define `model = neural_model()` and train the network using `model.fit`, the output will be the same, regardless of how many different times I ask for an output, and regardless of any random seed that is provided to Python.

   TRUE FALSE (circle one)

(c) (2 points) For classifying images from the MNIST dataset, we found in class that convolutional neural networks outperform a single layer neural network.

TRUE FALSE (circle one)

(d) (2 points) For classifying images from the facial images dataset (which we brought into Python with the command `from sklearn.datasets import fetch_lfw_people` ), we found in class that convolutional neural networks outperform a two layer neural network.

TRUE FALSE (circle one)

2. Suppose we run the commands

```
import pandas as pd
data = {
    "state": ["Ohio", "Ohio", "Ohio", "Nevada", "Nevada", "Nevada"],
    "year": [2000, 2001, 2002, 2001, 2002, 2003],
    "pop": [1.5, 1.7, 3.6, 2.4, 2.9, 3.2]
}
frame = pd.DataFrame(data)
```

(a) (5 points) What is the output of the following commands?

```
frame2 = frame.reindex(index = [3, 2, 5])
frame2
```

(b) (5 points) What is the output of the following commands?

```
frame3 = frame2.set_index("year")
frame3
```

[Both questions were covered in class.]

3. (10 points) Suppose we have a DataFrame named `df2` that contains single season 2 point field goal percentages of basketball players, of the form

```
        player      percentage      year
0       Clark      .7              2015
1       Jones      .6              2010
2       Smith      .5              2009
...
```

Suppose we have a DataFrame named `df3` that contains single season 3 point field goal percentages of basketball players, of the form

```
        player      percentage      year
0       Hawkins    .7              2015
1       Clark      .6              2010
2       Jones      .5              2009
...
```

Write a Python program that finds a player with the largest sum of 2 point and 3 point field goal percentages, among all players whose names appear in both `df2` and `df3`. Also, explain in complete sentences why your program performs this task.

[This was a modified homework problem.]

4. (10 points) What is the output of the following program? Explain your reasoning.

```
import re
data = '''
"data-testid="bar-chart--results-bar" style="width:51%"
role="progressbar" aria-valuenow="51" class="jsx-4201391551
jsx-842384122 labeled-bar df white"><span data-testid=
"bar-chart--results-bar-percent" class="jsx-4201391551 jsx-842384122"
'''
search_string = r'jsx([\w-]{5})'

found_strings = re.findall(search_string, data)
found_strings
```

[This was repeated from the practice exam]

5. (10 points) Give an example showing that the singular value decomposition is not unique.

That is, find positive integers $m, n, p$ and find a real $m \times n$ matrix $A$, $m \times m$ orthogonal matrices $U, \widetilde{U}$, $n \times n$ orthogonal matrices $V, \widetilde{V}$ and $p \times p$ diagonal matrices $D, \widetilde{D}$ (with $p \leq \min(m, n)$ and with nonzero diagonal entries) such that

$$A = U \begin{pmatrix} D & 0 \\ 0 & 0 \end{pmatrix} V = \widetilde{U} \begin{pmatrix} \widetilde{D} & 0 \\ 0 & 0 \end{pmatrix} \widetilde{V},$$

and such that either: $U \neq \widetilde{U}$, or $V \neq \widetilde{V}$, or $D \neq \widetilde{D}$.

(Recall that an orthogonal $n \times n$ matrix $U$ satisfies $U^T U = U U^T = I$, where $I$ denotes the $n \times n$ identity matrix.)

(Recall also that $\begin{pmatrix} D & 0 \\ 0 & 0 \end{pmatrix}$ is an $m \times n$ matrix, i.e. it is $D$ with zero entries added to its right and bottom sides if necessary in order to make $\begin{pmatrix} D & 0 \\ 0 & 0 \end{pmatrix}$ an $m \times n$ matrix.)

[This was repeated from Exam 1]

(Scratch paper)