Please provide complete and well-written solutions to the following exercises.

Due August 25, 10AM PST, to be uploaded as a single PDF document to blackboard (under the Assignments tab).

Homework 1

Exercise 1. Download and install Matlab on your personal computer. Instructions for downloading and installing this software can be found here: https://software.usc.edu/matlab/ . If you have not done so already, you should create a Mathworks account, associated to your USC email address (https://www.mathworks.com/login). Once you have installed Matlab, you should then install the NCM package (available at the bottom of this page: https://www.mathworks.com/moler/chapters.html). Once the NCM package is installed, you can access some of its features by just typing `ncmgui` in the Matlab command line.

Exercise 2. As needed, refresh your knowledge of proofs and logic by reading the following document by Michael Hutchings: http://math.berkeley.edu/∼hutching/teach/proofs.pdf

Exercise 3. As needed, take the following quizzes on logic and set theory:

http://scherk.pbworks.com/w/page/14864234/Quiz%3A%20Logic
http://scherk.pbworks.com/w/page/14864241/Quiz%3A%20Sets

(These quizzes are just for your own benefit; you don't need to record your answers anywhere.)

Exercise 4. In Matlab, do the following:

- Perform the following operation, and report the result:
$$\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + 4 \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}.$$
- Plot the function $f(x) = x^3 + e^x$ for x values in the interval $[0, 3]$.
- Describe the output of the following program.

```
x=1
while x~=0
    x=x/2
end
```

Exercise 5. In Matlab, the logical value 0 represents a false statement, and the logical value 1 represents a true statement. For example, `3<5` evaluates to a logical 1, and `5<3` evaluates to a logical 0.

Matlab's logical operations include: `&` for logical and, `|` for logical or, `~` for logical negation. Matlab's relational operations include: `<` for less than, `<=` for less than or equal to, `==` for equality, `~=` for not equality.

- Compute the following expression by hand, and in Matlab:

  `( (2<3) & (4<2) ) | ~(4<8).`

- Describe the output of the following program.

```
x=1
while (x<5) & ~(x<-5)
    x=x+rand
end
```

- Logical operations also apply to vectors (where 1 denotes true, and 0 denotes false). Compute the following expression by hand, and in Matlab:

  `([ 0 1 0] & [1 1 0] ) | [0 0 1].`

Exercise 6. Using Matlab, verify that its random number generator agrees with the law of large numbers and central limit theorem. For example, the command `rand(1,10^7)` generates 10^7 samples of a number that is equally likely to have any value in the interval $(0, 1)$ [rounded to the nearest floating point value]. You should then average these values, using e.g. the `mean` command, and check how close the average is to $1/2$. Then, make a histogram of samples using the `hist` command, and check how close the histogram is to a Gaussian function of the form

$$t \mapsto \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma^2}}, \qquad \forall\, t \in \mathbf{R}.$$

(More specifically, examine the histogram of `(mean(rand(10^3,10^4)) - 1/2 )/sqrt(10^(-3))` with say 100 bins. What does the quantity `mean(rand(10^3,10^4))` represent? What are the dimensions of `mean(rand(10^3,10^4))` ?) (Which value of $\sigma > 0$ gives you the closest fit between the histogram and the Gaussian function?)

(It is okay if you just try a few σ values and then pick your favorite one. The last part of this question does not need an extremely precise answer. This question is just meant to explore the concept of a bell curve, rather than to choose the absolute best fit for your histogram.)

Comment. Your answer to the above exercise should include the histogram you generated. To save a figure file in Matlab, either use the menu selection of File, Save As, and then save as e.g. a PDF, or use the command `saveas(gcf,'filename','pdf')` to save the current Matlab figure as a PDF file called `filename.pdf`.

If you ever want to read a description about a built-in Matlab function, such as the `saveas` function, use e.g. the command `help saveas`.

Exercise 7 (Optional)**.** Read some articles about errors in numerical computing with some serious consequences such as

- the Pentium FDIV bug (wikipedia)
- assorted disasters

- failure to convert from metric to imperial