Digest 3

(A compilation of emailed homework questions, answered around Wednesday.)

Question. [Exercise 4] Let $g(x) = x^{2/3}$.

- Show that $g'(0)$ does not exist.
- If $a \neq 0$, find $g'(a)$, using the definition of the derivative.
- Show that $y = x^{2/3}$ has a vertical tangent line at $x = 0$.
- Demonstrate the vertical tangent line by graphing $y = x^{2/3}$.

(From a student): How do I find $g'(a)$ using the derivative definition?

Answer. You are asked to use the limit definition of the derivative. Let $f(x) = x^{2/3}$. We have

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h\to 0} \frac{(a+h)^{2/3} - a^{2/3}}{h}.$$

We now need to use an algebraic identity.

Recall that $c^3 - d^3 = (c-d)(c^2 + cd + d^2)$. Using $c = (a+h)^{2/3}$ and $d = a^{2/3}$, we get the formula $(a+h)^2 - a^2 = ((a+h)^{2/3} - a^{2/3})((a+h)^{4/3} + (a+h)^{2/3}a^{2/3} + a^{4/3})$. Then

$$\begin{aligned}\frac{(a+h)^{2/3} - a^{2/3}}{h} &= \frac{(a+h)^{2/3} - a^{2/3}}{h} \cdot \frac{(a+h)^{4/3} + (a+h)^{2/3}a^{2/3} + a^{4/3}}{(a+h)^{4/3} + (a+h)^{2/3}a^{2/3} + a^{4/3}} \\ &= \frac{(a+h)^2 - a^2}{h[(a+h)^{4/3} + (a+h)^{2/3}a^{2/3} + a^{4/3}]}\end{aligned}$$

You should now be able to use this formula to compute the limit as $h \to 0$.

Question. (From a student): Can we use calculators or cheat sheets on the exam?

Answer. No. No calculators. No cheat sheets.

Question. (From a student): I noticed that there is an extra section named 2.9: the formal definition of a limit on the textbook. The exercise problems in this section look unfamiliar by addressing on absolute values and asking something like "Prove rigorously that...". I am wondering do we need to know anything from this section for the mid -term or for the entire Math 31A course?

Answer. You do not need to know the rigorous definition of the limit for this course. And I will never ask you to formally prove anything.

Question. (From a student): Could you provide some exercises to practice for the exam?

Answer. Other than the practice exams which have already been provided, here are some problems particular to product rule, quotient rule, and chain rule. These are exercises from the textbook http://ocw.mit.edu/ans7870/resources/Strang/Edited/Calculus/Calculus.pdf.

Section 2.5, page 77 (pdf page 84)

Exercises: 5,14,19,23,24, 34(a,b,c), 39(abce)

Solution Manual (for odd problems): http://ocw.mit.edu/resources/res-18-001-calculus-online-textbook-spring-2005/instructor-s-manual/MITRES-18-001-manual2.pdf