Digest 9

(A compilation of emailed homework questions, answered around Wednesday.)

Question. [Exercise 2] Evaluate

$$\lim_{n\to\infty} \frac{1^5 + 2^5 + \cdots + n^5}{n^6},$$

by showing that the limit is $\int_0^1 x^5 dx$.

(From a student): Are we supposed to reverse-engineer the limit function or can we use the integral and work from there and derive the limit function?

Answer. Either approach is valid in this exercise.

Question. (From a student): If the integration of a function $\cos x$ has the limit from 0 to 6. Do we deem it as non-existent or are we supposed to convert the limit into radians?

Answer. For cosine or other trigonometric functions, unless otherwise stated, the default input is in radians. The cosine function is defined for any real number, so such an integral does exist.

Question. (From a student): I was looking through the course notes you uploaded and I have a few questions. For example 4.8 and 4.9, why did you select $x_{n-1} = (n-1)/n$?

Answer. In this Example, we have chosen a particular Riemann sum over the interval $[a, b] = [0, 1]$. That is, we choose $x_0 = 0$, $x_1 = 1/n$, $x_2 = 2/n$, etc. In general, $x_i = i/n$ for each $0 \leq i \leq n$. In particular, choosing $i = n - 1$, we have $x_{n-1} = (n-1)/n$. And $x_n = n/n = 1$. This particular choice of rectangles for the Riemann sum is nice to work with, since all of the rectangles have the same width. That is, $x_i - x_{i-1} = 1/n$ for each $1 \leq i \leq n$. There are many ways to pick the rectangles, but this way is kind of the nicest way to do it.

Question. (From a student): Could we also have some harder practice finals?

Answer. Sure. See below. As usual, these other exams might have slightly different material on them. (For example, some of these exams have 31B material on them, so I have tried to indicate where to skip this material.)

Exam (Skip 2c, and skip 8) (For 9a, the shaded region is the sector of the circle with endpoints $(0, 0)$, $(\sqrt{2}, 0)$ and $(1, 1)$.)

Solutions

Exam (Skip 1.7, 1.12, 1.13, 1.15, 1.20, 2.7, 3a.1, 3a.4, 3c, 4, 6, 9b, 10, 12b, 12c, 12d, 13a) (The solution of 9c is incorrect.)

Solutions

Exam (Skip 1.4, 1.6, 1.9, 1.15, 1.16, 1.17, 1.18, 1.20, 2.2, 3.1, 3.5, 3.10, 5, 6c, 7a, 7c, 10,11,12)

Solutions

Exam (Skip 4d, 6a, 6c) (For 6d, you should be able to get some integral, but don't try to evaluate the integral.)

Exam (Skip 2, 5a, 5c)