Digest 3

(A compilation of emailed homework questions, answered around Wednesday.)

Question. [Exercise 5] Using the Pythagorean Theorem, derive the following formula.

$$\cos(\sin^{-1}(x)) = \sqrt{1-x^2}.$$

(From a student): What does it mean to derive the function using Pythagorean theorem?

Answer. I am asking you to derive this formula, using the same method we used in class on Friday. In this procedure, we drew a right triangle and used the Pythagorean Theorem.

Question. (From a student): What do we have to memorize for this exam?

Answer. The short answer is, that you should know enough things such that you can do all of the previous homeworks without looking stuff up. However, there are a lot of things that you don't need to remember. And in general, you don't need to remember proofs. I present some proofs of things to help you remember what is true, and to understand why things are true, but generally proofs are not tested in math classes until the upper division classes.

For now, I don't expect you to memorize the half angle and double angle formulas. I don't expect you to memorize all of the derivatives of inverse trig functions, derivatives of inverse hyperbolic functions, and things like that. Though, I would expect you to be able to re-derive (and perhaps use) inverse derivative formulas for $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$, since we discussed in class (and on the homework) how to derive these formulas. I do expect you to know the derivatives of exponential and logarithm functions. You should remember L'Hôpital's rule. You should remember how to integrate various trigonometric integrals, e.g. involving products of sines and cosines. You should remember how to integrate by parts. You should remember how to do use trigonometric substitution. You should remember that $\cos^2(x)+\sin^2(x)=1$, and you should know the identities that are obtained from this one by dividing by $\cos^2(x)$ or by $\sin^2(x)$. (This is a nice example where remembering how to derive the formula $1+\tan^2(x)=\sec^2(x)$ is probably more effective than memorizing the formula.)

In general, I don't expect you to memorize random integral formulas. However, if that integral formula can be derived in a sensible way using integration by parts, then you might be expected to compute such an integral, since you are supposed to know how to integrate by parts. Anyway, this is not an exhaustive list of things you should and should not know, but I hope it gives a good idea of what I do and do not expect you to know. In general, I want to test your understanding of things rather than your ability to regurgitate information.

Question. (From a student): I am wondering about the formulas and equations we need to memorize for the exam. If this is too prying into what material will be on the test I

apologize, but the chapters we have learned so far involve the exponential growth formula, compound interest formula, terminal velocity formula (and it's solution or the equation (V' equals, as well as V equals), Newton's Law of Cooling, Logistic Growth model, Hyperbolic sine and cosine, as well as arc trig, and inverse derivatives. Should I be memorizing all of these or am I going about this the wrong way?

Answer. You can of course memorize whatever you want, but in general I would recommend understanding these things instead. For example, you could memorize that $\cosh' = \sinh$, or you could remember that to derive this formula, you just need to take the derivatives of exponentials that define cosh. You could memorize inverse trig derivatives, but in my opinion it is easier to remember how to derive them (since they are all derived in essentially the same way). The same goes for exponential growth and compound interest. Terminal velocity, Newton's Law of Cooling and logistic growth do not need to be memorized but I expect you to understand them, in the same way that you needed to understand them on the homework. You could memorize the inverse derivative formula, or you could remember how to derive it instead (which in my opinion is easier). And so on.

Question. (From a student): How did you study for math exams when you were in school?

Answer. A week before the exam, I would start reviewing my course notes and the textbook, brushing up especially on more confusing things or things I didn't understand well, and looking at the harder homework problems. I created my own summaries and study guides as needed (I think this is a nice thing to do, since it helps to synthesize the information). I then did some assorted easy/hard problems from the textbook. About 2-4 days before the exam I took some practice exams, which I thought were a good way of measuring what I did and did not know. I studied most intensely two days before the exam, filling in the gaps from the practice exams. The day before the exam, I didn't study at all, to save energy for the exam itself. On the day of the exam, I did some light review in the morning, but nothing too intense or taxing. I also liked drinking soda during exams since it kept me alert.