Please provide complete and well-written solutions to the following exercises.

Due May 8, at the beginning of class.

Assignment 6

Exercise 1. Compute the force on one side of a square plate of side length 2 meters, where the plate is submerged vertically in a tank of water, and one side of the square is tangent the surface of the water.

Exercise 2. Compute the force on one side of a circular plate of radius 3 meters, where the plate is submerged vertically in a tank of water, and the top of the circle is tangent to the surface of the water.

Exercise 3. Compute the Taylor polynomials T_2 and T_3 for the function $f(x) = \dfrac{1}{1+x^2}$ at $x = a = 1$.

Exercise 4. Compute the Taylor polynomials T_2 and T_3 for the function $f(x) = \dfrac{1}{1+x}$ at $x = a = 2$. Then, compute the Taylor polynomials T_2 and T_3 for the function f at $x = a = 1$.

Exercise 5. Let T_n be the nth Taylor expansion of the function $f(x) = \cos(x)$ at $x = a = 0$. Find n such that the following error bounds holds:

$$|\cos(.3) - T_n(.3)| \leq 10^{-7}.$$

Exercise 6. Let T_n be the nth Taylor expansion of the function $f(x) = \sin(x)$ at $x = a = 0$. Find n such that the following error bounds holds:

$$|\sin(.5) - T_n(.5)| \leq 2^{-53}.$$

Exercise 7. Let $C > 0$ and let $a < b$. Let f, g be functions such that $|f(x) - g(x)| \leq C$. Show that

$$\left| \int_a^b f(x)dx - \int_a^b g(x)dx \right| \leq C(b-a).$$

This was an important step used in our error bound for Taylor polynomials.