Please provide complete and well-written solutions to the following exercises.

Due May 15, at the beginning of class.

Assignment 7

Exercise 1. For each of the following sequences, determine whether or not the sequence converges. (You may need to use the Squeeze Theorem or monotonicity.) If the sequence converges, compute its limit as $n \to \infty$.

- $a_n = n/2^n$.
- $a_n = 1 + (-1)^n$.
- $a_n = 1/(0.9)^n$.
- $a_n = (\sin n)/n$.
- $a_n = \left(1 - \frac{1}{n}\right)^n$.
- $a_n = (1/n)^{1/(\ln n)}$.

Exercise 2. From class, we saw that the sequence $a_n = (-1)^n$ does not converge as $n \to \infty$. Similarly, the sequence $b_n = (-1)^{n+1}$ does not converge as $n \to \infty$. However, $a_n + b_n = 0$ for all n, so $a_n + b_n$ does converge as $n \to \infty$. Therefore, in this case, $\lim_{n\to\infty}(a_n + b_n)$ is not equal to $\lim_{n\to\infty} a_n$ plus $\lim_{n\to\infty} b_n$. Explain how this does not contradict the theorem (limit laws for sequences) which stated $\lim_{n\to\infty}(a_n + b_n) = \lim_{n\to\infty} a_n + \lim_{n\to\infty} b_n$.

Exercise 3. Determine whether or not the following series converge or diverge. If the series converges, find its sum.

- $\sum_{n=0}^{\infty} (\sqrt{2})^n$.
- $\sum_{n=0}^{\infty} \left(\frac{1}{\sqrt{2}}\right)^n$.
- $\sum_{n=0}^{\infty} \frac{\cos(n\pi)}{5^n}$.
- $\sum_{n=1}^{\infty} \frac{n^n}{n!}$.
- $\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$.

Exercise 4. Suppose a ball is dropped from a height of 4 meters. Each time the ball hits the ground after falling from a height of h meters, the ball rebounds to the height of $(3/4)h$ meters. Find the total distance the ball travels up and down. Then, find the total number of seconds that the ball is moving. (Hint: the formula $s = (4.9)t^2$ implies that $t = \sqrt{|s|/4.9}$.)