

Name: _____ USC ID: _____ Date: _____

Signature: _____. Discussion Section: _____

(By signing here, I certify that I have taken this test while refraining from cheating.)

Exam 1

This exam contains 7 pages (including this cover page) and 5 problems. Enter all requested information on the top of this page.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- You have 50 minutes to complete the exam, starting at the beginning of class.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this. Scratch paper appears at the end of the document.

| Problem | Points | Score |
|---------|--------|-------|
| 1 | 10 | |
| 2 | 15 | |
| 3 | 10 | |
| 4 | 10 | |
| 5 | 10 | |
| Total: | 55 | |

Do not write in the table to the right. Good luck!^a

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1. Label the following statements as TRUE or FALSE. You do not need to explain your answer. No partial credit will be given in this section.

(a) (2 points) Let x be a real number. Then $\cos^{-1}(\cos(x)) = x$.

TRUE FALSE (circle one)

(b) (2 points) Consider $f(x) = x^2$ with domain $[-1, 1]$. Then f has an inverse function.

TRUE FALSE (circle one)

(c) (2 points) Let f be a function of a real variable with inverse g . If x is in the domain of g and if $f'(g(x)) \neq 0$, then $g'(x) = \frac{1}{f'(g(x))}$.

TRUE FALSE (circle one)

(d) (2 points) Let $f(x) = e^x$ with domain $(-\infty, \infty)$. Then the inverse of f is the function $\ln(x)$ with domain $(0, \infty)$.

TRUE FALSE (circle one)

(e) (2 points) Let $\sin^{-1}(x)$ denote the inverse sine of x . Let $-1 < x < 1$. Then

$$\cos(\sin^{-1}(x)) = \sqrt{1 - x^2}$$

TRUE FALSE (circle one)

2. Compute the following limits.

(a) (5 points) $\lim_{x \rightarrow 0^+} \frac{\cos(x) - 1}{x^2}.$

(b) (5 points) $\lim_{x \rightarrow 0} \frac{x + 2}{x + 1}.$

(c) (5 points) $\lim_{x \rightarrow \infty} \tan^{-1}(x).$ (As usual, \tan^{-1} denotes the inverse of the tangent function.)

3. Evaluate the following integrals.

(a) (5 points) $\int x \ln(x) dx.$

(b) (5 points) $\int_4^5 \frac{1}{t \ln t} dt.$

4. Evaluate the following integrals.

(a) (5 points) $\int_0^{\pi/2} \cos^5(x) dx.$

(b) (5 points) $\int \frac{t}{t^2 - 3t + 2} dt.$

5. (10 points) Evaluate the following integral.

$$\int_{\sqrt{2/3}}^{\sqrt{2}} x^{-4} \sqrt{2+x^2} \, dx.$$

(Scratch paper)