Math 126, Fall 2019, USC Instructor: Steven Heilman

Name: _____ USC ID: _____ Date: _____

Signature: _____. Discussion Section: _____
(By signing here, I certify that I have taken this test while refraining from cheating.)

Exam 2

This exam contains 7 pages (including this cover page) and 5 problems. Enter all requested information on the top of this page.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- You have 50 minutes to complete the exam, starting at the beginning of class.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this. Scratch paper appears at the end of the document.

Do not write in the table to the right. Good luck![a]

Problem	Points	Score
1	12	
2	10	
3	13	
4	15	
5	15	
Total:	65	

Some Formulas:
$\frac{d}{dx} \ln|\sec(x) + \tan(x)| = \sec(x)$.

[a] November 7, 2019,

1. Circle exactly one answer for each multiple choice question. You do not need to show any work in this section. No partial credit will be given in this section.

   (a) (3 points) Suppose we estimate the integral of the function $f(x) = x^2$ on the interval $[0, 1]$ using the trapezoid rule with the points $0 < 1/4 < 1/2 < 3/4 < 1$. That is, we approximate the area under the curve of f by four trapezoids, whose endpoints are $0, 1/4, 1/2, 3/4, 1$ in succession. Then the trapezoid rule gives the estimate T_4, where T_4 is equal to

   (i) $\frac{1}{4}\big((1/4)^2 + (1/2)^2 + (3/4)^2 + 1/2\big)$.

   (ii) $\frac{1}{4}\big((1/8)^2 + (3/8)^2 + (5/8)^2 + (7/8)^2\big)$.

   (iii) $\frac{1}{4}\big((1/4)^2 + (1/2)^2 + (3/4)^2 + 1^2\big)$.

   (iv) $\frac{1}{4}\big((1/4)^2/2 + (1/2)^2 + (3/4)^2 + 1^2/2\big)$.

   (v) None of the above.

   (b) (3 points) Let $f(x) = \sin(x)$. The third order Taylor polynomial of f centered at the point $a = 0$ is

   (i) $x - (1/3)x^3$.

   (ii) $x - (1/6)x^3$.

   (iii) $x - x^3$.

   (iv) x.

   (v) None of the above.

   (c) (3 points) Suppose a square is submerged in a fluid with density ρ. The square is oriented vertically in the fluid, and its edges are two meters long. The top edge of the square touches and is parallel to the top of the fluid. Let g be the standard acceleration due to gravity. The fluid force on one side of the square is:

   (i) $\rho g \int_0^2 h dh$.

   (ii) $\rho g \int_0^2 h^2 dh$.

   (iii) $\rho g \int_0^2 2h^2 dh$.

   (iv) $\rho g \int_0^2 2h dh$.

   (v) None of the above.

   (d) (3 points) If $\{a_n\}$ is a divergent sequence, then

   (i) $\{a_n\}$ is bounded. (There is some $c > 0$ such that $|a_n| \leq c$ for all $n \geq 0$)

   (ii) $\{a_n\}$ is not bounded.

   (iii) $\{a_n\}$ is monotonic. (The sequence is increasing, or it is decreasing.)

   (iv) $\{1/a_n\}$ converges.

   (v) None of the above need to be true.

2. Evaluate the following integrals. Denote whether or not the integral converges.

(a) (5 points) $\int_0^1 \frac{dx}{x^2-1}$

Circle One: | The Integral Converges | The Integral Diverges |

(b) (5 points) $\int_0^{10} \frac{xdx}{\sqrt{1+x^2}}$.

Circle One: | The Integral Converges | The Integral Diverges |

3. (13 points) Compute the arc length of the function $f(x) = -\ln(\cos(x))$ from $x = 0$ to $x = \pi/4$.

Your Answer:

4. (a) (5 points) Compute $\lim_{n\to\infty}\left(\sqrt{n+1}-\sqrt{n}\right)$.

Your Answer:

(b) (5 points) Determine whether or not the following series converges or diverges. If the series converges find its sum.

$$\sum_{n=0}^{\infty}(\sqrt{2})^n.$$

Circle One: Converges Diverges

(c) (5 points) Determine whether or not the following series converges or diverges. If the series converges find its sum.

$$\sum_{n=1}^{\infty}\ln\left(\frac{n}{n+1}\right).$$

Circle One: Converges Diverges

5. (a) (5 points) Using integrals, write a quantity that computes the area between the curves $x = y^3 + 3y + 1$ and $x = 4y + 1$. You do NOT have to evaluate the integrals. You MUST integrate with respect to the variable y.

Your Answer:

(b) (5 points) Suppose we take the region between the curves $x = y^2 - 3y$ and $x = 2y - y^2$ and we revolve this region around the line $x = 3$. Write an integral that computes the volume of the resulting solid region. You do NOT have to evaluate the integral. You MUST use the disk method (volume by revolution).

Your Answer:

(c) (5 points) Suppose we take the curve $y = x^2 + 5$ where $1 \leq x \leq 2$ and we revolve this curve around the y axis. Write an integral that computes the **surface area** of the resulting solid region. You do NOT have to evaluate the integral.

Your Answer:

(Scratch paper)