Math 126, Fall 2019, USC **Circle One item below**:

Pawlowski 9AM Pawlowski 10AM Ndaoud 11AM Dreyer 12PM Heilman 1PM

Name: ____________________ USC ID: ____________ Date: __________

Signature: ____________________.
(By signing here, I certify that I have taken this test while refraining from cheating.)

Final Exam

This exam contains 18 pages (including this cover page) and 8 problems. Enter all requested information on the top of this page.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- You have 120 minutes to complete the exam.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the scratch paper that appears at the end of the document. **Clearly indicate any reference to your scratch paper.**

Do not write in the table to the right. Good luck![a]

Some Formulas:
$\int \sec^3(x)dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(x)+\tan(x)| + C$. $\cos^2(x) = \dfrac{1+\cos(2x)}{2}$. $\sin^2(x) = \dfrac{1-\cos(2x)}{2}$.

Problem	Points	Score
1	30	
2	20	
3	25	
4	25	
5	30	
6	20	
7	20	
8	25	
Total:	195	

1. Each row in the following table corresponds to a multiple choice question below. **For each row in the table, circle exactly one corresponding answer.**

Question	Answer				
(a)	(i)	(ii)	(iii)	(iv)	(v)
(b)	(i)	(ii)	(iii)	(iv)	(v)
(c)	(i)	(ii)	(iii)	(iv)	(v)
(d)	(i)	(ii)	(iii)	(iv)	(v)
(e)	(i)	(ii)	(iii)	(iv)	(v)
(f)	(i)	(ii)	(iii)	(iv)	(v)

All of your answers must be given in the box above. Any answers written below this sentence will not be graded.

You do not need to show any work in this section. No partial credit will be given in this section.

(a) (5 points) The function $f(x) = \frac{1}{2}\ln(x+1) + e^x$ is one-to-one (that is, f passes the horizontal line test) (you do not need to verify that fact). Find the equation of the tangent line to $y = f^{-1}(x)$ at the point $x = 1$.

(i) $y = \dfrac{2}{3}x - 1.$

(ii) $y = \dfrac{2}{3}x - \dfrac{2}{3}.$

(iii) $y = 2x - 3.$

(iv) $y = \dfrac{3}{2}x + 1.$

(v) $y = \dfrac{1}{2}x + \dfrac{1}{2}.$

(b) (5 points) The length of the function $f(x) = \dfrac{x^3}{6} + \dfrac{1}{2x}$, with $x \in [\frac{1}{2}, 1]$ is given by:

(i) $\dfrac{1}{2}\displaystyle\int_{1/2}^{1} \sqrt{1 + (x^2 + x^{-2})^2}\, dx.$

(ii) $\dfrac{1}{2}\displaystyle\int_{1/2}^{1} \sqrt{1 + (x^1 + x^{-1})^2}\, dx.$

(iii) $\dfrac{1}{2}\displaystyle\int_{1/2}^{1} (x^1 + x^{-1})\, dx.$

(iv) $\dfrac{1}{2}\displaystyle\int_{1/2}^{1} (x^2 + x^{-2})\, dx.$

(v) $\dfrac{1}{2}\displaystyle\int_{1/2}^{1} \sqrt{x^2 + x^{-2}}\, dx.$

(c) (5 points) Consider the following series

$$\text{(I)} \sum_{n=3}^{+\infty} n \ln\left(1+\frac{1}{n}\right) \qquad \text{(II)} \sum_{n=10}^{+\infty} \frac{(-1)^n}{\sqrt{n^2+3n+2}}.$$

Which one of the following statements is true?

(i) (I) is divergent and (II) is conditionally convergent.

(ii) (I) is absolutely convergent and (II) diverges.

(iii) (I) and (II) are both conditionally convergent.

(iv) (I) is absolutely convergent and (II) is conditionally convergent.

(v) (I) and (II) both diverge.

(d) (5 points) $\displaystyle\int_{-1}^{0} \frac{1}{x^2-4x+3}\,dx$ is equal to

(i) $(1/2)\ln(3) + (1/2)\ln(2) - (1/2)\ln(4)$.

(ii) $(1/2)\ln(3) - (1/2)\ln(2) - (1/2)\ln(4)$.

(iii) $\ln(3) - \ln(8)$.

(iv) $\ln(3) + \ln(8)$.

(v) We cannot specify a finite value. The integral diverges.

(e) (5 points) Which of the following is $\displaystyle\int \frac{f'(x)}{g(x)}\,dx$ equal to?

(i) $\displaystyle\frac{f(x)}{g(x)} - \int \frac{f(x)}{(g(x))^2}\,dx.$

(ii) $\displaystyle\frac{f'(x)}{g(x)} - \int \frac{f(x)}{(g(x))^2}\,dx.$

(iii) $\displaystyle\frac{f(x)}{g(x)} + \int \frac{f(x)}{(g(x))^2}\,dx.$

(iv) $\displaystyle\frac{f(x)}{g(x)} + \int \frac{f'(x)}{(g(x))^2}\,dx.$

(v) none of the above

(f) (5 points) Which of the following integrals converges?

(i) $\displaystyle\int_1^\infty \frac{x^5-1}{x^4+x^2+3}\,dx.$

(ii) $\displaystyle\int_1^\infty \frac{x^4}{x^5+1}\,dx.$

(iii) $\displaystyle\int_1^\infty \frac{x}{x^3-8}\,dx.$

(iv) $\displaystyle\int_1^\infty \frac{x}{x^3+8}\,dx.$

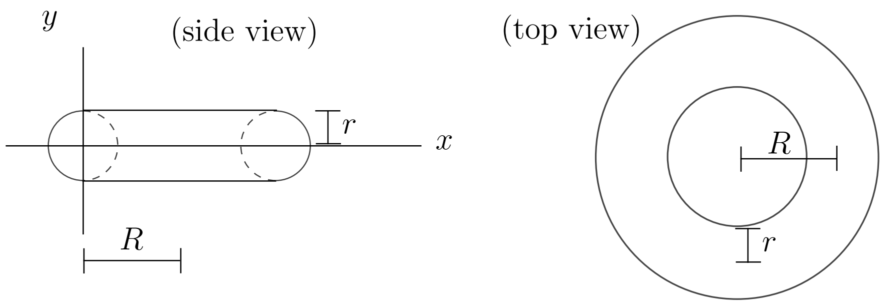
(v) none of the above.

2. Consider the surface S obtained by rotating the curve $y = e^{-x}$ about the x–axis, with $0 \leq x < \infty$.

   (a) (10 points) Establish that the **surface area** of S is given by $A = 2\pi \displaystyle\int_0^1 \sqrt{1 + u^2}\, du$.

   (b) (10 points) Calculate the value of the integral A.

Your Answer:



: Consider a torus (donut) of major radius R and minor radius r $(0 < r < R)$. The torus is defined by rotating a disc $x^2 + y^2 \leq r^2$ of radius r around the axis $x = R$.

3. (a) (10 points) Using the method of **cylindrical shells**, show that the volume of the torus is given by the integral

$$V = 4\pi R \int_{-r}^{r} \sqrt{r^2 - x^2}\, dx - 4\pi \int_{-r}^{r} x\sqrt{r^2 - x^2}\, dx.$$

(b) (5 points) Calculate the value of $J = \int_{-r}^{r} x\sqrt{r^2 - x^2}\, dx$.

Your Answer:

(c) (10 points) Calculate the value of $I = \int_{-r}^{r} \sqrt{r^2 - x^2}\, dx$. Deduce the value of V.

Your answer: | $I =$ | $V =$ |

4. Consider $f(x) = \dfrac{2}{3-x}$ with domain $[0, 2]$. Note that, the range of f is contained in $[0, 2]$. (You do not need to show this fact and may use it freely.)

   Let $u_0, u_1, u_2, \ldots$ be the sequence defined by $u_0 = \dfrac{3}{2}$ and $u_{n+1} = f(u_n)$, for all $n \geq 0$.

   (a) (5 points) Show that, for any $n \geq 0$, we have $0 \leq u_n \leq 2$.

   You **must** use a mathematical induction to prove this assertion.

   (b) (10 points) Show that the sequence $u_0, u_1, u_2, \ldots$ is decreasing. You **must** use mathematical induction to prove this assertion.

   Hint: Use the fact that f is increasing.

(c) (10 points) Show that the sequence $u_0, u_1, u_2, \ldots$ converges to a limit ℓ, then determine the value of ℓ. You must carefully justify your answer.

$\ell =$

5. (a) (10 points) Determine whether or not the following series converges. **Show all of your work**.

$$\sum_{n=1}^{\infty} \frac{3^{(n^2)}}{n!}.$$

Circle One:

Converges	Diverges

(b) (10 points) Determine whether the following series converges absolutely, converges conditionally, or diverges. **Show all of your work**.

$$\sum_{n=1}^{\infty} \frac{\cos(n)}{n^2}.$$

Circle One:

Converges Absolutely	Converges Conditionally	Diverges

(c) (10 points) Let $0 < r < s$ be constants. Does the sequence $\{(r^n + s^n)^{1/n}\}$ converge as $n \to \infty$? If it does converge, what is the limit?

Your Answer:

6. (a) (10 points) Suppose a particle is moving back and forth along the x-axis. The particle begins at the point $x = 0$. It then moves in the positive x-direction a distance of 1. The particle then moves in the negative x-direction a distance of $1/4$. The particle then moves in the positive x-direction a distance of $1/16$. The particle then moves in the negative x-direction a distance of $1/64$, and so on. In general, whenever the particle moves a distance h in one direction, it then moves in the opposite direction a distance of $h/4$. What position on the x-axis does the particle eventually approach?

Your Answer:

(b) (10 points) Evaluate

$$\sum_{n=0}^{\infty} \int_{n}^{n+1} xe^{-x} dx.$$

Your Answer:

7. (20 points) For the following power series, find the radius of convergence R. Describe the set of all points where the power series converges absolutely, describe the set of all points where the power series converges conditionally, and describe the set of all points where the power series diverges.

$$\sum_{n=1}^{\infty}(n+1)x^{n-1}.$$

Lastly, evaluate the sum (that is, simplify the sum so that no $\sum$ appears in your final answer.)

Your Answer:

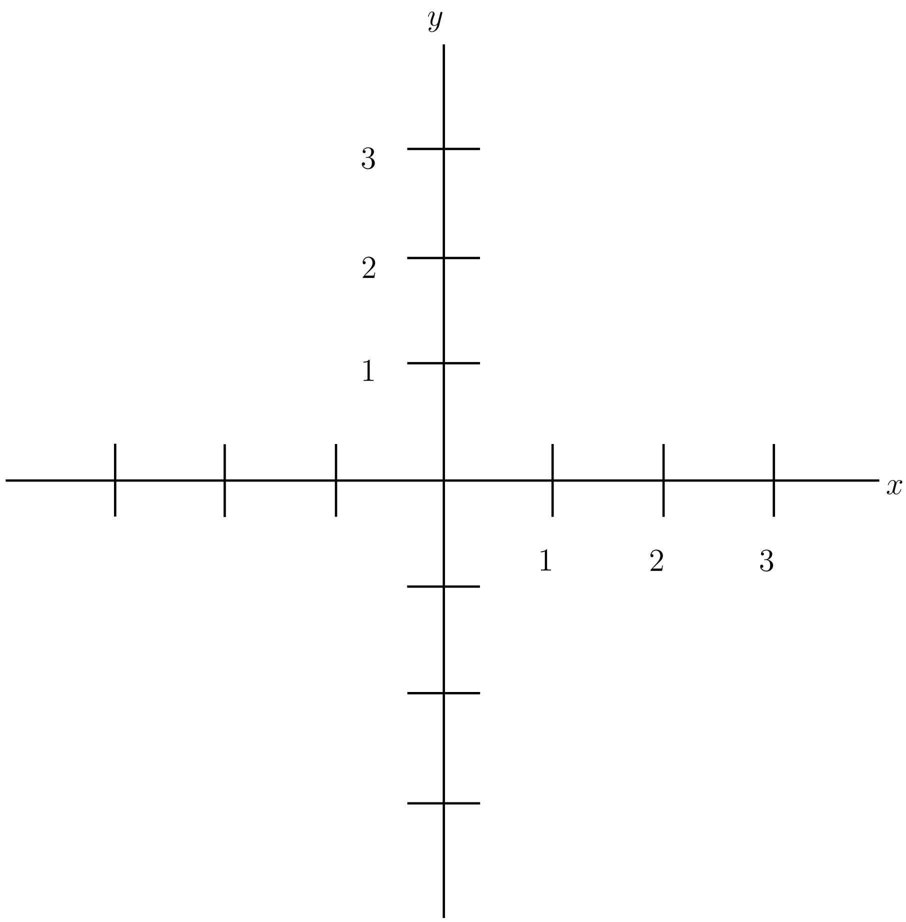
Radius of Convergence:	Converges Absolutely on:	Converges Conditionally on:	Diverges on:

The Sum is Equal to:

8. Consider the equation

$$r = \sin(4\theta).$$

(a) (5 points) In the figure below, draw the polar curve $r = \sin(4\theta)$ for all $0 \leq \theta \leq 2\pi$.



(b) (10 points) Find the area enclosed by the curve, in the first quadrant. (The first quadrant is the region where $x \geq 0$ and $y \geq 0$).

Your Answer:

(c) (5 points) Find the slope of the tangent line to the curve when $\theta = \pi/4$.

Your Answer:

(d) (5 points) Write an integral that computes the arc length of this curve where $0 \leq \theta \leq \pi/4$. You do NOT have to evaluate this integral.

Your Answer:

(Scratch paper 1)

(Scratch paper 2)

(Scratch paper 3)