Please provide complete and well-written solutions to the following exercises.

Due October 10, at the beginning of class.

Assignment 6

Exercise 1. Let $r, h > 0$ be constants. Let $f(x) = \frac{r}{h}x$, where $0 \leq x \leq h$. Compute the volume by revolution of f over $0 \leq x \leq h$, where f is rotated around the x-axis. You should find a formula for the volume of a circular cone of radius r and height h. (Hint: use the disk method.)

Exercise 2. Compute the volume of the solid obtained by revolving the region enclosed by the ellipse $x^2 + 9y^2 = 9$ about the x-axis.

Exercise 3. Consider the region in the plane contained between the lines $y = x$, $y = 0$, $x = 0$ and $x = 1$. Compute the volume obtained by rotating this region around the y-axis.

Exercise 4. Suppose we take the region between the curves $x = y^2 - 3y$ and $x = 2y - y^2$ and we revolve this region around the line $x = 3$. Write an integral that computes the volume of the resulting solid region. You do NOT have to evaluate the integral.

Exercise 5. Find the arc length of $y = 2^{-4}x^4 + (1/2)x^{-2}$ over the interval $[1, 4]$. (Hint: write $1 + (y')^2$ as the square of something.)

Exercise 6. Using a comparison of integrals, show that the arc length of $y = x^{4/3}$ over $[1, 2]$ is greater than or equal to $5/3$.

Exercise 7. Compute the force on one side of a square plate of side length 2 meters, where the plate is submerged vertically in a tank of water, and one side of the square is tangent the surface of the water.

Exercise 8. Compute the force on one side of a circular plate of radius 3 meters, where the plate is submerged vertically in a tank of water, and the top of the circle is tangent to the surface of the water.