Please provide complete and well-written solutions to the following exercises.

Due January 9, at the beginning of class.

Assignment 1

Exercise 1. Estimate the double integral

$$\iint_{[0,2]\times[0,2]} f dA.$$

To do this, use a Riemann sum with four terms in the sum which represent four squares of area 1 each. Use also the lower left point of each square to compute the Riemann sum. Use the following facts: $f(0,0) = 1$, $f(0,1) = 2$, $f(1,0) = 5$, and $f(1,1) = -1$.

Exercise 2. Calculate the following integral.

$$\int_{x=0}^{x=1} \int_{y=0}^{y=1} 15xy\sqrt{x^2+y^2}\, dydx.$$

Exercise 3. Sketch the region of integration of the following integral, and then evaluate the integral.

$$\int_{x=0}^{x=\pi} \int_{y=0}^{y=x} x \sin y\, dydx.$$

Exercise 4. Sketch the region of integration of the following integral, and then reverse the order of integration to get an equivalent integral.

$$\int_{y=0}^{y=1} \int_{x=y}^{x=\sqrt{y}} dxdy.$$

Exercise 5. Reverse the order of integration and evaluate the integral. Explain why the reversed order of integration is simpler to evaluate.

$$\int_{y=0}^{y=1} \int_{x=y}^{x=1} \frac{\sin x}{x} dxdy.$$

Exercise 6. Integrate the function $f(x,y) = (x+y+1)^{-2}$ over the triangle with vertices $(0,0)$, $(4,0)$ and $(0,6)$.

Exercise 7. Integrate the function $f(x,y) = x^2 y$ over the region $1 \leq x \leq 2$ and $x \leq y \leq 2x+2$.

Exercise 8. Sketch the region of integration, reverse the order of integration, and then evaluate the integral.

$$\int_{x=0}^{x=3} \int_{y=x/3}^{1} e^{y^2} dydx.$$

Exercise 9. Find the area of the region bounded by the parabola $x = y - y^2$ and the line $y = -x$.

Exercise 10. Find the volume of the solid figure in Euclidean space $\mathbf{R}^3$ that lies in the column $|x| + |y| \leq 1$, and between the planes $z = 0$ and $3x + z = 3$.

Exercise 11. Find the volume of the solid figure in Euclidean space $\mathbf{R}^3$ bounded above by the paraboloid $z = x^2 + y^2$ and bounded below by the triangle in the plane $z = 0$, where the triangle is itself bounded by the planes $y = x$, $x = 0$ and $x + y = 2$.

Exercise 12. Let D denote the region in the plane bounded by the three curves $y = 0$, $y = (x + 1)^2$ and $x = y - y^3$, where $x \geq -1$ and $y \leq 1$. Evaluate

$$\iint_D y dA.$$