Department of Teaching and Learning, Steinhardt School of Culture, Education, & Human Development
Department of Mathematics, Courant Institute of Mathematical Sciences
New York University

MTHED-UE-1049: Mathematical Proof and Proving (MPP)
MATH-UA-125: Introduction to Mathematical Proofs

Homework No. 10

This homework should be submitted just before the beginning of class, on April 23rd, 2012.
You should bring to class a copy of the homework that you submit, or at least notes that can remind you of what you did, in order to participate in class discussions.
You must justify all answers and all steps in your proofs.

1. Following your class investigation of the Towers of Hanoi, answer the following questions and provide a proof for each answer:
   1.1 Why can we be sure that for any natural number *n*, it is possible to move a tower of *n* rings from one needle to another?
   1.2 How many steps are needed to move a tower of *n* rings from one needle to another?
   1.3 How many years (approximately) would it take to move a tower of 64 rings from one needle to another, if every day exactly one ring is moved.

2. What is the sum of the interior angles of an *n*-sided convex polygon? Prove your claim.

3. The first terms of a sequence are given as: $a_1 = 2$, $a_2 = 4$, $a_3 = 8$, $a_4 = 16$, $a_5 = 32$.
   3.1 Can you find a rule for a_n?
   3.2 Can you find a another rule, different than the one you found in 3.1?
   3.3 Based on your responses to 3.1 and 3.2, what can you say about the sequence a_n? Is it uniquely defined?

4. A sequence is defined (explicitly) by: $a_n = 10^n - 7$, $\forall n \in N$.
   4.1 Prove that if a_n is divisible by 9, then a_{n+1} is also divisible by 9.
   4.2 Is a_n divisible by 9, for any natural number *n*?