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MTHED-UE-1049: Mathematical Proof and Proving (MPP)
MATH-UA-125: Introduction to Mathematical Proofs

Homework No. 9

This homework should be submitted just before the beginning of class, on April 16th, 2012.
You should bring to class a copy of the homework that you submit, or at least notes that can remind you of what you did, in order to participate in class discussions.

1. A sequence is defined (explicitly) by $a_n = \frac{n\cdot(n+1)}{2}$, $\forall n \in N$.

   What is a_{n+1}? a_{n-1}? a_{n+5}? a_{2n-1}? Simplify the expressions you get.

2. A sequence is defined (explicitly) by $a_n = \frac{3^{2n-1}}{4^n}$, $\forall n \in N$.

   What is a_{n+1}? a_{n-1}? a_{n+5}? a_{2n-1}? Simplify the expressions you get.

3. A sequence is defined (explicitly) by $\sigma_n = \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \cdots + \frac{1}{2n}$, $\forall n \in N$.

   3.1 What is σ_{n+1}? σ_{n+2}?

   3.2 Find: $\sigma_{n+1} - \sigma_n$.

4. A sequence is defined recursively by: (*i*) $a_1 = 1$ and (*ii*) $a_n = 3\cdot a_{n-1}$, $\forall n \in N$.

   Conjecture a formula for a_n and verify that your formula is correct.

5. A sequence is defined recursively by: (*i*) $b_1 = 3$ and (*ii*) $b_n = 3\cdot b_{n-1}$, $\forall n \in N$.

   Conjecture a formula for b_n and verify that your formula is correct.

6. Are the two sequences defined in problems 4 and 5 (above) the same? Explain your answer.

7. Based on what we did in class, write a proof of the following statement:
   $\forall n \in N, 1+3+5\cdots(2n-1) = n^2$.
   Make sure that you write the Given and the RTP, and that you explain all steps.