Please provide complete and well-written solutions to the following exercises.

Due April 23, in the discussion section.

Assignment 3

Exercise 1. Which of the following mappings are linear transformations from $\mathbf{R}^2$ to $\mathbf{R}^2$?

(a) $(x, y) \mapsto (3x - y + 1, -x)$.
(b) $(x, y) \mapsto (y, x)$.
(c) $(x, y) \mapsto (2x, y)$.

Exercise 2. Section 2.2, Exercise 1(bcdf) in the textbook.

Exercise 3. Section 2.2, Exercise 2(aefg) in the textbook.

Exercise 4. Section 2.2, Exercise 7 in the textbook.

Exercise 5. Let V, W be finite-dimensional vector spaces over a field $\mathbf{F}$ such that $\dim(V) = \dim(W)$. Let $T\colon V \to W$ be linear. Prove: if T is onto, then T is one-to-one.

Exercise 6. Let V be a (possibly infinite-dimensional) vector space over a field $\mathbf{F}$. Let $S\colon V \to \mathbf{F}$ and let $T\colon V \to \mathbf{F}$ be linear transformations. Assume that $N(S) \supseteq N(T)$. Show that there exists $\alpha \in \mathbf{F}$ such that $S = \alpha T$.

Exercise 7. Prove that the field $\mathbf{R}$ of real numbers is infinite-dimensional, if we consider $\mathbf{R}$ as a vector space over the field $\mathbf{Q}$.