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MTHED-UE-1049: Mathematical Proof and Proving (MPP)
MATH-UA-125: Introduction to Mathematical Proofs

Practice Exam

To be submitted by May 7th 2012 at 11:45am

There are 9 questions and some sub-questions on this practice exam. You should communicate your answers in a clear and coherent way, maintaining mathematical accuracy and validity. Please write your full name on the top of each page. All the writing should be done on the pages that are part of this exam.

Your grade on this practice exam will be taken into account for your final course grade, only in your favor. **You will not be penalized if your grade on the practice exam is low**. We encourage you to do your best to answer all the questions. This experience will help prepare you for final exam.

Your work will be graded only if you meet the above deadline for submission. If you do not submit the practice exam before the beginning of next class, it will not be checked nor will it be graded. You should solve the problems on your own. Additionally, we will not grade papers that appear too similar to each other.

Next class will be devoted mostly to reviewing the practice exam problems, as well as other questions you may have. You will benefit the most if you come prepared to articulate any difficulties you encountered, and if you **come to class with a copy of this practice exam and your answers to use during this class time**.

1. Prove that $|x - 1| + |x + 5| \geq 6$ for all $x \in R$.

2. Prove that if $n \in N, n \geq 3$, then $5^n + 6^n < 7^n$.

3. Consider the following statement:

"If c is less than 20, then the equation $x^2 + 8x + c = 0$ has 2 real roots."

i. Give an example that satisfies the statement. Explain why it satisfies it.
ii. Give an example that contradicts the statement. Explain why it contradicts it.

4. Prove or disprove the following statement:

   For every $n \in \mathbb{N}$, $(3n^2 + 3n + 23)$ is a prime number.

5. Consider the following statement, denoted by (*):

(*) For every $n \in \mathbb{N}, 1+2+3+\cdots+n=\frac{n^2+n+2}{2}$

Study carefully the following proof for the above statement and answer the subsequent questions.

Given:

(*) is true for $n=k$, that is:

$$1+2+3+\cdots+k=\frac{k^2+k+2}{2}$$

RTP:

(*) is true for $n=k+1$

PROOF:

$$1+2+3+\cdots+k+(k+1) \underbrace{=}_{\substack{Based \\ on\ the\ Given}} \frac{k^2+k+2}{2}+(k+1)$$

$$=\frac{k^2+k+2+2(k+1)}{2}=\frac{k^2+2k+1+3}{2}=\frac{(k+1)^2+(k+1)+2}{2}$$

Q.E.D.

(*i*) Choose the best response for the above proof:

a. The proof shows that statement (*) is always true

b. The proof shows that statement (*) is true in some cases

c. The proof shows that statement (*) is always false.

d. The proof does not show anything about the truth-value of statement (*).

(*ii*) Justify your answer.

6. Consider the following statement:

For every y where $y \in Z$, there exists an x where $x \in Z$, such that $x^2 = y$

For each of the given numbers, determine whether it constitutes a counterexample. Justify your claims.

(a) $y = 4$
(b) $y = 0.25$
(c) $y = -9$
(d) $y = -0.16$

7.

(a) Prove that the product of any 4 consecutive integers is always a multiple of 4.

(b) Let $x \in \mathbb{Z}$, Prove that if $(x-1)(x+1)(x+2)$ is not divisible by 4, then x is divisible by 4.

8. Prove that for all natural numbers n, $4^n + 15n - 1$ is divisible by 9.

9. Prove that $\sqrt{5}$ is an irrational number.