Math 167, Spring 2016, UCLA Instructor: Steven Heilman

Name: ______________________ UCLA ID: ______________ Date: ___________

Signature: ______________________.
(By signing here, I certify that I have taken this test while refraining from cheating.)

Mid-Term 2

This exam contains 8 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page.

You may *not* use your books, notes, or any calculator on this exam. You are required to show your work on each problem on this exam. The following rules apply:

- You have 50 minutes to complete the exam, starting at the beginning of class.
- **If you use a theorem or proposition from class or the notes or the book you must indicate this** and explain why the theorem may be applied. It is okay to just say, "by some theorem/proposition from class."
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this. Scratch paper appears at the end of the document.

Do not write in the table to the right. Good luck![a]

Problem	Points	Score
1	15	
2	15	
3	10	
4	15	
5	15	
Total:	70	

Reference sheet

Below are some definitions that may be relevant.

$$\Delta_m := \{x = (x_1, \ldots, x_m) \in \mathbf{R}^m \colon \sum_{i=1}^{m} x_i = 1,\, x_i \geq 0,\ \forall\, 1 \leq i \leq m\}.$$

Let m, n be positive integers. Suppose we have a two-player general sum game with $m \times n$ payoff matrices. Let A be the payoff matrix for player I and let B be the payoff matrix for player II. A pair of vectors $(\widetilde{x}, \widetilde{y})$ with $\widetilde{x} \in \Delta_m$ and $\widetilde{y} \in \Delta_n$ is a **Nash equilibrium** if

$$\widetilde{x}^T A \widetilde{y} \geq x A \widetilde{y}, \quad \forall\, x \in \Delta_m,$$

$$\widetilde{x}^T B \widetilde{y} \geq \widetilde{x} B y, \quad \forall\, y \in \Delta_n.$$

A joint distribution of strategies is an $m \times n$ matrix $z = (z_{ij})_{1 \leq i \leq m, 1 \leq j \leq n}$ such that $z_{ij} \geq 0$ for all $i \in \{1, \ldots, m\}$, $j \in \{1, \ldots, n\}$, and such that

$$\sum_{i=1}^{m} \sum_{j=1}^{n} z_{ij} = 1.$$

We say z is a **correlated equilibrium** if

$$\sum_{j=1}^{n} z_{ij} a_{ij} \geq \sum_{j=1}^{n} z_{ij} a_{kj}, \qquad \forall\, i \in \{1, \ldots, m\}, \forall\, k \in \{1, \ldots, m\}.$$

$$\sum_{i=1}^{m} z_{ij} b_{ij} \geq \sum_{i=1}^{m} z_{ij} b_{ik}, \qquad \forall\, j \in \{1, \ldots, n\}, \forall\, k \in \{1, \ldots, n\}.$$

1. (15 points) Recall the prisoner's dilemma, which has the following payoffs.

		Prisoner II	
		silent	confess
Prisoner I	silent	$(-1, -1)$	$(-10, 0)$
	confess	$(0, -10)$	$(-8, -8)$

Find all Correlated equilibria for this game.

2. (15 points) Find the value of the two-person zero-sum game described by the payoff matrix

$$\begin{pmatrix} 1 & 3 & 3 & 4 \\ 4 & 3 & 3 & 1 \\ 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 4 \end{pmatrix}$$

3. (10 points) Prove the case $d = 1$ of Sperner's Lemma: Suppose the unit interval $[0, 1]$ is partitioned such that $0 = t_0 < t_1 < \cdots < t_n = 1$, where each t_i is marked with a 1 or 2 whenever $0 < i < n$, t_0 is marked 1 and t_n is marked 2. Then the number of ordered pairs (t_i, t_{i+1}), $0 \leq i < n$ with different markings is odd.

4. For all questions below, **justify your answer**.

(a) (5 points) Give an example of a closed and convex subset K of Euclidean space, and give an example of a continuous function $f\colon K \to K$ such that f has no fixed point.

(b) (5 points) Give an example of a bounded and closed subset K of Euclidean space, and give an example of a continuous function $f\colon K \to K$ such that f has no fixed point.

(c) (5 points) Give an example of a function $f\colon \mathbf{R} \to \mathbf{R}$ such that the only fixed point of f is the point $x = 1$.

5. (15 points) Let A be an $m \times n$ real matrix. Prove:

$$\min_{y \in \Delta_n} \max_{x \in \Delta_m} x^T A y = \min_{y \in \Delta_n} \max_{i=1,\ldots,m} (Ay)_i.$$

(Scratch paper)